Module 11: Standard Deviations and the Like

This module describes measures of dispersion or unlikeness, including standard deviations, variances, ranges and sums of squares.

Knowing the Mean is not Enough

- What else would it be useful to know?
- A key issue is how alike or “unlike” each other the individual observations are
- How can we measure “unlikeness”

Standard Deviations and the Like

- Measures of Unlikeness
- Measures of Dispersion
- Measures of Variation

How to measure unlikeness?

Samples from Module 10

Sample 1	Sample 2
x_1	x_2
18	90
19	4
20	3
21	2
22	1

Sample 1

Pair	Difference
18, 19	-1
18, 20	-2
18, 21	-3
18, 22	-4
19, 20	-1
19, 21	-2
19, 22	-3
20, 21	-1
20, 22	-2
21, 22	-1
Total	

Calculating Sums of Squares

SAMPLE 2 DATA

Age	Deviation from Mean	Square of Deviation from Mean
x	$(x-\bar{x})$	$(x-\bar{x})^2$
1	$(1 - 20) = -19$	$(-19)^2 = 361$
2	$(2 - 20) = -18$	$(-18)^2 = 324$
3	$(3 - 20) = -17$	$(-17)^2 = 289$
4	$(4 - 20) = -16$	$(-16)^2 = 256$
90	$(90 - 20) = 70$	$(70)^2 = 4{,}900$
$\sum x = 100$	$\sum (x-\bar{x}) = 0$	$\sum (x-\bar{x})^2 = 6{,}130$

Sum of Squared Deviations from the Mean

$$\sum (x - \bar{x})^2 = \text{Sum of Squares (SS)}$$

$$= SS(x)$$

$$= 6{,}130$$

Calculating s^2 for Sample 2

$$s^2 = \frac{SS(x)}{n-1} = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$= \frac{6{,}130}{4} = 1{,}532.5 \text{ years}^2$$

$$s = \text{SD} = \sqrt{1532.5} = 39.15 \text{ years}$$

Calculating SS(x) for Sample 1

Age	Deviation from Mean	Square of Deviation from Mean
x	$(x - \overline{x})$	$(x - \overline{x})^2$
18	$(18 - 20) = -2$	$(-2)^2 = 4$
19	$(19 - 20) = -1$	$(-1)^2 = 1$
20	$(20 - 20) = 0$	$(0)^2 = 0$
21	$(21 - 20) = 1$	$(1)^2 = 1$
22	$(22 - 20) = 2$	$(2)^2 = 4$
$\sum x = 100$	$\sum (x - \overline{x}) = 0$	$\sum (x - \overline{x})^2 = 10$

Calculating s^2 for Sample 1

$$SS(x) = \sum (x - \bar{x})^2 = 10$$

$$s^2 = \frac{SS(x)}{n-1} = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$= \frac{10}{4} = 2.5 \text{ years}^2$$

$$s = SD = \sqrt{2.5} = 1.58 \text{ years}$$

Easier Method of Computing SS(x)

Age x	$(\text{Age})^2$ x^2
18	$18^2 = 324$
19	$19^2 = 361$
20	$20^2 = 400$
21	$21^2 = 441$
22	$22^2 = 484$
$\sum x = 100$	$\sum x^2 = 2{,}010$

Calculating s^2

$$SS(x) = \sum (x - \bar{x})^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$SS(x) = 2,010 - \frac{100^2}{5} = 10$$

$$s^2 = \frac{SS(x)}{n-1} = \frac{10}{4} = 2.5$$

Measures of Central Tendency and Variation

	Sample 1	Sample 2
Mean	20 years	20 years
Median	20 years	3 years
Mode	None	None
Midrange	20 years	45.5 years
Variance	2.5 years^2	1,532.5 years^2
Standard Deviation	1.58 years	39.15 years
Range	4 years	89 years

Population Parameters vs. Sample Statistics

Population Parameters (Truth)	Sample Statistics (Estimates or guesses)
Mean (μ)	Mean ($\bar{x}$)
Variance (σ^2)	Variance (s^2)
Standard deviation (σ)	SD, standard deviation (s)

TEST

A random sample of n = 5 persons was selected from the population of persons at day care centers in a large city. The ages of the n = 5 persons were: 4, 3, 4, 4, 20.

For this sample determine:

Mean ($\overline{x}$)

Median

Midrange

Mode

Range

Variance (s^2)

Standard deviation (s)