Module 18: One Way ANOVA

This module begins the process of using variances to address questions about means. Strategies for more complex study designs appear in a subsequent module.

Reviewed 11 May 05 /MODULE 18

Independent Random Samples from Two Populations of Serum Uric Acid values

	Sample 1	Sample 2
	1.2	1.7
	0.8	1.5
	1.1	2.0
	0.7	2.1
	0.9	1.1
	1.1	0.9
	1.5	2.2
	0.8	1.8
	1.6	1.3
	0.9	1.5
Sum	10.6	16.1
Mean	1.06	1.61

Serum Acid SS (Total) Worksheet

Person	x	x^2
1	1.2	1.44
2	0.8	0.64
3	1.1	1.21
4	0.7	0.49
5	0.9	0.81
6	1.1	1.21
7	1.5	2.25
8	0.8	0.64
9	1.6	2.56
10	0.9	0.81
11	1.7	2.89
12	1.5	2.25
13	2.0	4.00
14	2.1	4.41
15	1.1	1.21
16	0.9	0.81
17	2.2	4.84
18	1.8	3.24
19	1.3	1.69
20	1.5	2.25
Sum	26.7	39.65
Mean	1.34	
Sum^2/n	35.64	
SS(Total)	4.01	
Variance	0.21	
SD	0.46	

SS (Within) and SS (Among) worksheet

	x	x^2	x	x^2
	1.2	1.44	1.7	2.89
	0.8	0.64	1.5	2.25
	1.1	1.21	2.0	4.00
	0.7	0.49	2.1	4.41
	0.9	0.81	1.1	1.21
	1.1	1.21	0.9	0.81
	1.5	2.25	2.2	4.84
	0.8	0.64	1.8	3.24
	1.6	2.56	1.3	1.69
	0.9	0.81	1.5	2.25
Sum	10.6	12.06	16.1	27.59
Mean	1.06		1.61	
Sum^2/n	11.236		25.921	
SS	0.824		1.669	
Variance	0.092		0.185	
SD	0.303		0.431	

SS (Within) = SS (sample 1) + SS (sample 2)

= 0.824 + 1.669
= 2.490

SS (Within) = 2.49

$$SS(Among) = \frac{sum_1^2}{n_1} + \frac{sum_2^2}{n_2} - \frac{total^2}{20}$$

$$= \frac{(10.6)^2}{10} + \frac{(16.1)^2}{10} - \frac{(26.7)^2}{20}$$

$$= 11.236 + 25.921 - 35.64$$

$$= 1.51$$

SS(Among) = 1.51

1. The hypothesis: H_0: $\mu_1 = \mu_2$ vs H_1: $\mu_1 \neq \mu_2$

2. The assumptions: Independent random samples , normal distributions, $\sigma_1^2 = \sigma_2^2$

3. The α-level : $\alpha = 0.05$

4. The test statistic: ANOVA

5. The rejection region: Reject H0: $\mu_1 = \mu_2$ if

$$F = \frac{MS(Among)}{MS(Within)} > F_{0.95(1,18)} = 4.41$$

Where MS(Among)=SS(Among)/DF(Among)

MS(Within)=SS(Within)/DF(Within)

6. The result:

ANOVA				
Source	df	SS	MS	F
Among	1	1.52	1.52	10.86
Within	18	2.49	0.14	
Total	19	4.01		

7. The conclusion: Reject H_0: $\mu_1 = \mu_2$

Since $F = 10.86 > F_{0.95}(1,18) = 4.41$

Testing the Hypothesis that the Two Serum Uric Acid Populations have the Same Mean

1. The hypothesis: $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

2. The α-level: $\alpha = 0.05$

3. The assumptions: Independent Random Samples
Normal Distribution $\sigma_1^2 = \sigma_2^2$

4. The test statistic:

$$t = \frac{\overline{x}_1 - \overline{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5. The reject region: Reject H_0 if t is not between ± 2.1009

6. The result:

$$t = \frac{0.55}{0.37(0.45)} = 3.30$$

7. The conclusion: Reject $H_0 : \mu_1 = \mu_2$ since t is not between ± 2.1009

Example 2

Independent Random Samples from Three Populations of Serum Uric Acid Values

	Sample 1	Sample 2	Sample 3
	1.2	1.7	1.3
	0.8	1.5	1.5
	1.1	2.0	1.4
	0.7	2.1	1.0
	0.9	1.1	1.8
	1.1	0.9	1.4
	1.5	2.2	1.9
	0.8	1.8	0.9
	1.6	1.3	1.9
	0.9	1.5	1.8
Sum	10.6	16.1	14.9
Mean	1.06	1.61	1.49

Independent Random Samples from Three Populations of Serum Uric Acid Values

ANOVA Worksheet

	1		2		3		Combined Total	
	x	x^2	x	x^2	x	x^2	x	x^2
	1.2	1.44	1.7	2.89	1.3	1.69		
	0.8	0.64	1.5	2.25	1.5	2.25		
	1.1	1.21	2.0	4.00	1.4	1.96		
	0.7	0.49	2.1	4.41	1.0	1.00		
	0.9	0.81	1.1	1.21	1.8	3.24		
	1.1	1.21	0.9	0.81	1.4	1.96		
	1.5	2.25	2.2	4.84	1.9	3.61		
	0.8	0.64	1.8	3.24	0.9	0.81		
	1.6	2.56	1.3	1.69	1.9	3.61		
	0.9	0.81	1.5	2.25	1.8	3.24		
Sum	10.6	12.06	16.1	27.59	14.9	23.37	41.6	63.020
n	10		10		10		30	
Mean	1.06		1.61		1.49		1.39	
Sum^2/n	11.236		25.921		22.201		57.685	
SS	0.824		1.669		1.169		5.335	
Variance	0.092		0.185		0.130		0.184	
SD	0.303		0.431		0.360		0.429	

$$SS(\text{Among}) = 11.236 + 25.921 + 22.201 - 57.685$$

$$= 1.673$$

$$SS(\text{Within}) = 0.824 + 1.669 + 1.169$$

$$= 3.662$$

$$SS(\text{Total}) = 1.673 + 3.662 = 5.335$$

Testing the Hypothesis that the Three populations have the same Average Serum Uric Acid Levels

1. **The hypothesis:** H_0: $\mu_1=\mu_2=\mu_3$, vs. H_1: $\mu_1 \neq \mu_2 \neq \mu_3$

2. **The assumptions:** Independent random samples
   normal distributions $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$

3. **The α-level :** $\alpha = 0.05$

4. **The test statistic:** ANOVA

5. The Rejection Region: Reject $H_0: \mu_1 = \mu_2 = \mu_3$ if

$$F = \frac{MS(\text{Among})}{MS(\text{Within})} > F_{0.95(2,27)} = 3.35$$

where

$$MS(\text{Among}) = \frac{SS(\text{Among})}{df\,(\text{Among})}, \quad MS(\text{Within}) = \frac{SS(\text{Within})}{df\,(\text{Within})}$$

6. The Result:

ANOVA

Source	df	SS	MS	F
Among	2	1.67	0.84	6.00
Within	27	3.66	0.14	
Total	29	5.33		

7. The Conclusion: Reject H_0: Since $F = 6.00 > F_{0.95}\ (2, 27) = 3.35$.

Example 3

A random sample of n = 10 was taken from each of three populations of young males. Systolic blood pressure measurements were taken on each child. The measurements are listed below.

	Group		
	1	2	3
	100	104	105
	102	88	112
	96	100	90
	106	98	104
	110	102	96
	110	92	110
	120	96	98
	112	100	86
	112	96	80
	90	96	84
Sum	1,058	972	965
Mean	105.8	97.2	96.5

Independent Random Samples from Three Populations of Blood Pressure Levels

ANOVA Worksheet

	1		2		3		Combined Total	
	x	x^2	x	x^2	x	x^2	x	x^2
	100	10,000	104	10,816	105	11,025		
	102	10,404	88	7,744	112	12,544		
	96	9,216	100	10,000	90	8,100		
	106	11,236	98	9,604	104	10,816		
	110	12,100	102	10,404	96	9,216		
	110	12,100	92	8,464	110	12,100		
	120	14,400	96	9,216	98	9,604		
	112	12,544	100	10,000	86	7,396		
	112	12,544	96	9,216	80	6,400		
	90	8,100	96	9,216	84	7,056		
Sum	1,058	112,644	972	94,680	965	94,257	2,995	301,581
n	10		10		10		30	
Mean	105.8		97.2		96.5		99.8	
Sum^2/n	111,936		94,478		93,123		299,001	
SS	708		202		1135		2580	
Variance	78.6		22.4		126.1		89.0	
SD	8.9		4.7		11.2		9.4	

SS(Among) = 111,936 + 94,478 + 93,123 - 299,001

= 536.47

SS(Within) = 708 + 202 + 1,134

= 2,043.70

SS(Total) = 536 + 2,043 = 2,580.17

Testing the Hypothesis That the Three Populations Have the Same Average Blood Pressure Levels

1. The hypothesis: $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_2 \neq \mu_3$

2. The assumptions: Independent random samples
normal distributions $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$

3. The α-level : α = 0.05

4. The test statistic: ANOVA

5. The Rejection Region: Reject H_0: $\mu_1 = \mu_2 = \mu_3$ if

$$F = \frac{\text{MS(Among)}}{\text{MS(Within)}} > F_{0.95(2,27)} = 3.35$$

where

$$\text{MS(Among)} = \frac{\text{SS(Among)}}{df\,\text{(Among)}}, \quad \text{MS(Within)} = \frac{\text{SS(Within)}}{df\,\text{(Within)}}$$

6. The Result:

ANOVA

Source	DF	SS	MS	F
Among	2	536.47	268.23	3.54
Within	27	2043.70	75.69	
Total	29	2580.17		

7. The Conclusion: Reject H_0: $\mu_1 = \mu_2 = \mu_3$, since

$$F = 3.54 > F_{0.95}(2, 27) = 3.35$$

	Group			
	1	2	3	
	100	104	105	
	102	88	112	
	96	100	90	
	106	98	104	
	110	102	96	
	110	92	110	
	120	96	98	
	112	100	86	
	112	96	80	
	90	96	84	
				Total
$\bar{x}$	105.8	97.2	96.5	$99.83 = \bar{\bar{x}}$
$\bar{x} - \bar{\bar{x}}$	+5.97	-2.63	-3.33	---

Group Effect (arrows point to the values in the $\bar{x} - \bar{\bar{x}}$ row)

For Group 1, first child,
Individual effect = $x_{11} - \overline{x}_1$ = 100 - 105.8 = - 5.8

Individual Value	=	Overall Mean	+	Group Effect	+	Individual Effect

100 = 99.83 + 5.97 + (-5.80)

$$X_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

Group Effect   Random Effect

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

Pulmonary Function Equipment Comparison

A calibration evaluation of four machines that measure pulmonary function yielded, with the four machines being located at four sites,

Machine /Site

1	2	3	4
NC	Jackson	Minn	Balt
433	445	434	441
435	440	436	443
432	438	433	438
439	441	437	439
436		434	442
		438	444
		440	
		435	

The numbers recorded above each represent one replication and are a computer generated count that is supposed to be equivalent to one liter. A difference of 1% or more is not acceptable. Consider the following questions:

1. Is there evidence that the four machines are not equally calibrated?

ANOVA

Source	df	SS	MS	F
Among	3	177.25	59.08	9.22
Within	19	121.71	6.41	
Total	22	298.96		

	NC	Jack	Minn	Balt	All Four
	433	445	434	441	
	435	440	436	443	
	432	438	433	438	
	439	441	437	439	
	436		434	442	
			438	444	
			440		
			435		
Sum(x)	2,175	1,764	3,487	2,647	10,073
n	5	4	8	6	23
Mean	435.0	441.0	435.9	441.2	438.0
Sum(x^2)	946,155	777,950	1,519,935	1,167,795	4,411,835
SS	30.00	26.00	38.88	26.83	298.96
Variance	7.50	8.67	5.55	5.37	13.59
SD	2.74	2.94	2.36	2.32	3.69
$(Sum(x))^2/n$	946,125.0	777,924.0	1,519,896.1	1,167,768.2	4,411,536.0

SS(Total)	298.96
SS(Among)	177.25
SS(Within)	121.71

Example: *AJPH, Sept. 1997; 87 : 1437*

TABLE 2—Comparison of Emotional States and Psychosocial Adjustment among Patients in Cardiopulmonary Resuscitation (CPR) Treatment Groups and Control Group across Time (Mean Scores ± SD)

	Control Group (n = 99)	CPR– Social Support Group (n = 74)	CPR– Education Group (n = 74)	CPR-Only Group (n = 90)
Anxiety (range, 0–21)				
Baseline	6.3 ± 4.7	6.1 ± 4.7	7.3 ± 4.6	6.6 ± 4.6
2 wk	6.9 ± 4.6	5.8 ± 4.8	7.1 ± 4.7	7.0 ± 4.9
3 mo	5.8 ± 4.2	5.6 ± 4.3	7.2 ± 4.7	7.3 ± 4.6
6 mo[a]	5.6 ± 4.1	5.2 ± 4.6	7.2 ± 4.8	7.4 ± 4.9
Depression (range, 0–40)				
Baseline	11.8 ± 7.0	12.4 ± 6.4	11.8 ± 5.5	12.9 ± 7.1
2 wk	12.5 ± 7.8	11.3 ± 7.4	11.8 ± 6.2	13.2 ± 7.9
3 mo	11.4 ± 6.5	11.8 ± 7.3	12.1 ± 5.6	13.3 ± 7.1
6 mo	11.0 ± 6.4	11.3 ± 7.2	12.2 ± 5.9	13.5 ± 8.0
Hostility (range, 0–30)				
Baseline	7.4 ± 4.3	8.1 ± 4.5	8.4 ± 4.2	8.7 ± 4.8
2 wk	8.2 ± 4.4	7.6 ± 4.4	8.3 ± 4.0	8.7 ± 5.0
3 mo	7.5 ± 4.5	7.6 ± 4.2	8.6 ± 3.9	8.8 ± 5.2
6 mo[b]	7.2 ± 4.4	7.0 ± 4.4	8.4 ± 4.1	9.3 ± 5.8
Psychosocial adjustment to illness (range, 0–100)[c]				
Baseline	42.6 ± 10.0	40.0 ± 9.4	42.8 ± 1.9	45.4 ± 12.5
3 mo[d]	41.6 ± 10.5	39.0 ± 9.9	41.5 ± 10.2	45.2 ± 12.9
6 mo[e]	41.3 ± 9.2	38.2 ± 9.0	40.6 ± 9.5	45.4 ± 13.3

Note. Data were collected before family members attended CPR training, then 2 weeks, 3 months, and 6 months following CPR training. Family members in the control group did not attend CPR training.

[a] $P = .004$ for univariate analysis of variance (ANOVA); for post hoc comparisons, $P = .03$ for CPR-only group vs CPR–social support group, $P = .04$ for CPR-only group vs control group.

[b] $P = .007$ for univariate ANOVA; $P = .02$ for CPR-only group vs CPR–social support group, $P = .02$ for CPR-only group vs control group.

[c] Higher scores indicate poorer adjustment.

[d] $P = .02$ for univariate ANOVA; $P = .005$ for CPR-only group vs CPR–social support group, no significant differences for other group comparisons.

[e] $P = .003$ for univariate ANOVA; $P = .001$ for CPR-only group vs CPR–social support group, $P = .03$ for CPR-only group vs CPR–education group, $P = .03$ for CPR-only group vs control group.

ANOVA for Anxiety at Baseline

	Control	CPR Social	CPR Education	CPR Only	Total
n	99	74	74	90	337
mean	6.3	6.1	7.3	6.6	
SD	4.7	4.7	4.6	4.6	
Sum	623.70	451.40	540.20	594.00	2,209.30
Sum^2/n	3,929.31	2,753.54	3,943.46	3,920.40	14,546.71
S^2	22.09	22.09	21.16	21.16	
SS	2,164.82	1,612.57	1,544.68	1,883.24	7,205.31
$Total^2/337$	14,483.70				

ANOVA

	df	SS	MS	F
Among	3	63.01	21.00	0.97
Within	333	7,205.31	21.64	
Total	336	7,268.32		

$P > 0.05$

$F_{0.95}(3,333) = 2.60$　　$F_{0.99}(3,333) = 3.78$

$F_{0.975}(3,333) = 3.12$　　$F_{0.995}(3,333) = 4.25$

ANOVA for Psychosocial Adjustment to illness at 3 months

	Control	CPR Social	CPR Education	CPR Only	Total
n	99	74	74	90	337
mean	41.6	39.0	41.5	45.2	
SD	10.5	9.9	10.2	12.9	
Sum	4,118.40	2,886.00	3,071.00	4,068.00	14,143.40
Sum^2/n	171,325.44	112,554.00	127,446.50	183,873.60	595,199.54
S^2	110.25	98.01	104.04	166.41	
SS	10,804.50	7,154.73	7,594.92	14,810.49	40,364.64
$Total^2/337$	593,577.93				

ANOVA

	df	SS	MS	F
Among	3	1,621.61	540.54	4.46
Within	333	40,364.64	121.22	
Total	336	41,986.25		

$P < 0.005$

$F_{0.95}(3,333) = 2.60$ $F_{0.99}(3,333) = 3.78$

$F_{0.975}(3,333) = 3.12$ $F_{0.995}(3,333) = 4.25$

Example: *AJPH, August 2001; 91:1258*

Heart Rate as a Predictor of Mortality: The MATISS Project

Fulvia Seccareccia, MSc, Fabio Pannozzo, MD, Francesco Dima, Anna Minoprio, MSc, Antonio Menditto, MD, Cinzia Lo Noce, and Simona Giampaoli, MD

Objectives. This study sought to verify the independent role of heart rate in the prediction of all-cause, cardiovascular, and noncardiovascular mortality in a low-risk male population.

Methods. In an Italian population-based observational study, heart rate was measured in 2533 men, aged 40 to 69 years, between 1984 and 1993. Data on cardiovascular risk factors were collected according to standardized procedures. Vital status was updated to December 1997.

Results. Of 2533 men followed up (representing 24 457 person-years), 393 men died. Age-adjusted death rates for 5 heart rate levels showed increasing trends. The adjusted hazard rate ratios for each heart rate increment were 1.52 (95% confidence interval [CI] = 1.29, 1.78) for all-cause mortality, 1.63 (95% CI = 1.26, 2.10) for cardiovascular mortality, and 1.47 (95% CI = 1.19, 1.80) for noncardiovascular mortality. Relative risks between extreme levels were more than 2-fold for all endpoints considered.

Conclusions. Heart rate is an independent predictor of cardiovascular, noncardiovascular, and total mortality in this Italian middle-aged male population. (*Am J Public Health.* 2001;91:1258–1263)

TABLE 2—Characteristics of Study Population, by Heart Rate Class: MATISS Project, 1984–1997

	Heart Rate, Beats per Minute						
	<60 (n = 642)	60–69 (n = 836)	70–79 (n = 477)	80–89 (n = 181)	≥90 (n = 97)	F	P
Systolic blood pressure, mm Hg, mean ± SD	140.3 ± 20.0	141.5 ± 19.6	143.5 ± 20.1	148.4 ± 23.9	150.5 ± 25.8	10.07	.000
No. of cigarettes smoked per day, mean ± SD	6.6 ± 10.1	7.6 ± 10.4	8.0 ± 10.9	8.6 ± 12.1	7.9 ± 10.6	1.97	.096
Body mass index, kg/m^2, mean ± SD	27.3 ± 3.6	27.3 ± 3.7	27.6 ± 3.6	27.2 ± 3.8	28.0 ± 4.1	1.40	.232
Serum cholesterol level, mg/dL, mean ± SD	219.9 ± 40.3	222.5 ± 42.4	224.3 ± 43.8	233.1 ± 48.5	229.3 ± 44.9	4.01	.003
Forced expiratory volume, mL/m^2, mean ± SD	953 ± 190	943 ± 189	913 ± 185	855 ± 220	876 ± 187	13.14	.000
Arm circumference, cm, mean ± SD	27.3 ± 2.9	27.3 ± 2.8	27.2 ± 3.0	27.2 ± 2.7	27.2 ± 2.8	0.15	.962
Prevalence of diabetes, %	4.3	3.4	7.1	7.8	9.3	15.94[a]	.003

[a]Pearson χ^2.

Systolic Blood Pressure, mm Hg, within Heart Rate Categories
Table 2, MATISS Project

	Heart Beats per Minute					
	< 60	60 - 69	70 - 79	80 - 89	> 90	Total
n	642	836	477	181	97	2,233
Mean	140.3	141.5	143.5	148.4	150.5	
SD	20	19.6	20.1	23.9	25.8	

Systolic Blood Pressure, mm Hg, within Heart Rate Categories
Table 2, MATISS Project

	Heart Beats per Minute					
	< 60	60 - 69	70 - 79	80 - 89	> 90	Total
n	642	836	477	181	97	2,233
Mean	140.3	141.5	143.5	148.4	150.5	
SD	20	19.6	20.1	23.9	25.8	
Sum	90,072.6	118,294.0	68,449.5	26,860.4	14,598.5	318,275.0
Sum^2/n	12,637,185.8	16,738,601.0	9,822,503.3	3,986,083.4	2,197,074.3	45,364,521.1
SS	256,400.0	320,773.6	192,308.8	102,817.8	63,901.4	

SS(Within) = 256,400.0 + … + 63,901.4 = 936,201.6

SS(Among) = 12,637,185.8 + … + 2,197,074.3 - 45,364,521.1
= 16,926.5

ANOVA

Source	df	SS	MS	F
Among	4	16,926.5	4,231.63	10.07
Within	2,228	936,201.6	420.20	
Total	2,232	953,128.1		