

ANOVA				
Source	df	SS	MS	F
Drugs	2	37.44	18.72	8.22
Exercise	2	14.11	7.06	3.10
Interaction	4	10.22	2.56	1.12
Treatments	8	61.78	7.72	
Within Cells	9	20.50	2.28	
Total	17	82.28		

$$F(\text{Interaction}) = 1.12 \quad F_{0.95}(4, 9) = 3.63$$

$$F(\text{Drugs}) = 8.22 \quad F_{0.95}(2, 9) = 4.26$$

$$F(\text{Exercise}) = 3.10 \quad F_{0.95}(2, 9) = 4.26$$

Module 31: ANOVA for Factorial Designs

This module discusses analyses for factorial designs.

Factorial Designs

Factorial designs include two or more factors, each having more than one level or treatment. Participants typically are randomized to a combination that includes one treatment or level from each factor.

Treatment Combinations

Treatment combinations and the ability to assess interaction are the essence of factorial designs.

Drug and Exercise Factorial Example

In this two factor example, participants received one of four drugs and also participated in one of four exercise programs. Hence, each factor has four levels and there are 16 treatment combinations altogether. Three participants were assigned to each of the 16 treatment combinations, so that a total of $n = 48$ participants were involved.

An activity intensity score was recorded for each participant.

Activity response score

Drug	Type of Exercise				Total
	A	B	C	D	
1	4	2	9	3	
	3	2	7	4	
	7	1	3	3	
Total	14	5	19	10	48
2	2	5	5	5	
	4	3	2	4	
	6	7	3	7	
Total	12	15	10	16	53
3	3	4	7	9	
	6	8	5	5	
	5	5	4	6	
Total	14	17	16	20	67
4	2	9	4	9	
	2	4	6	5	
	3	5	5	8	
Total	7	18	15	22	62
Overall	47	55	60	68	230

Advantage of Factorial Designs

This design permits an assessment of the presence of interaction between the two factors and, if no interaction is present, of the effect of the different levels of exercise and of the different drugs.

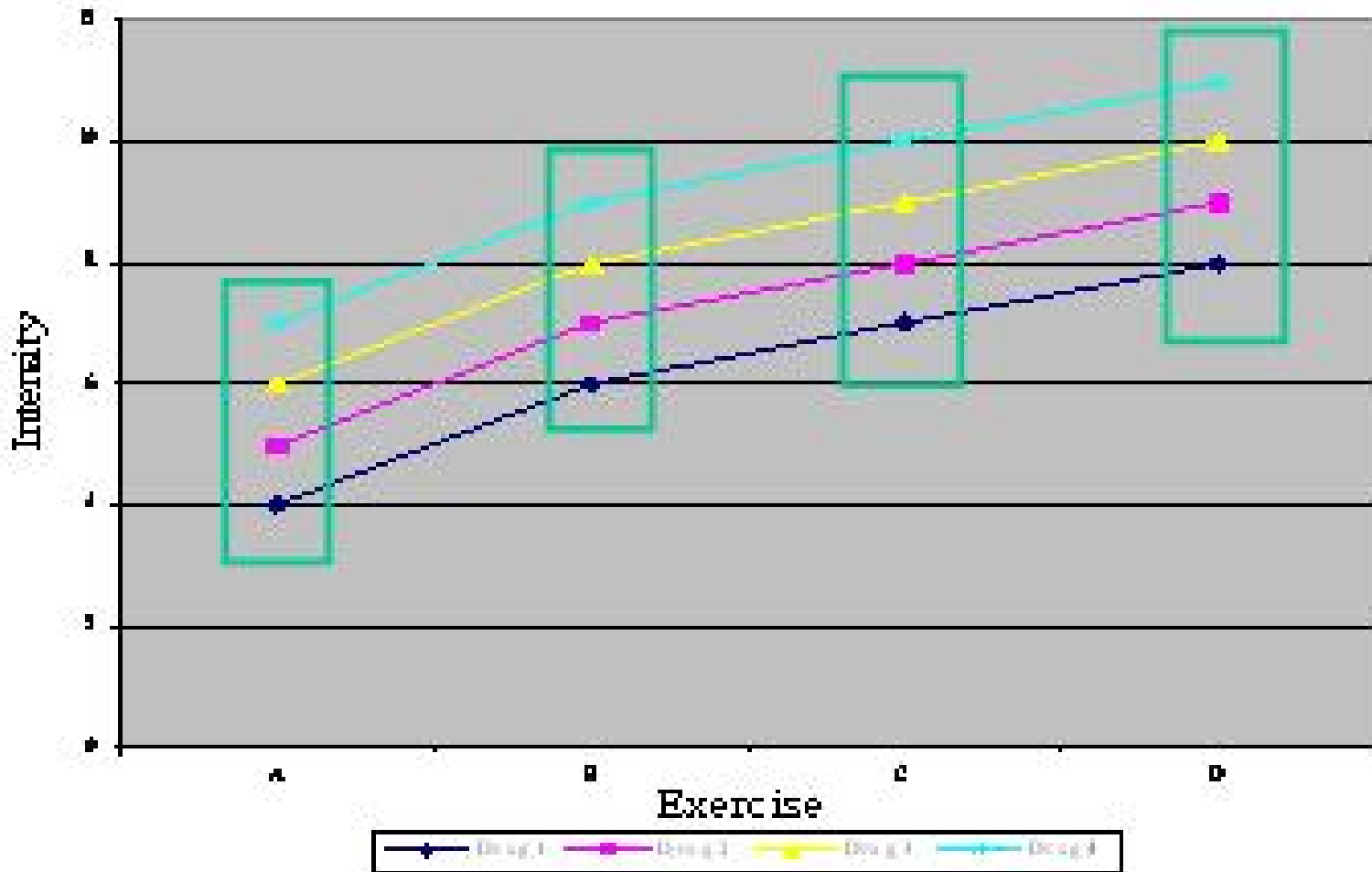
The design is more efficient than two separate studies, one for each of the two factors.

Essence of Interaction

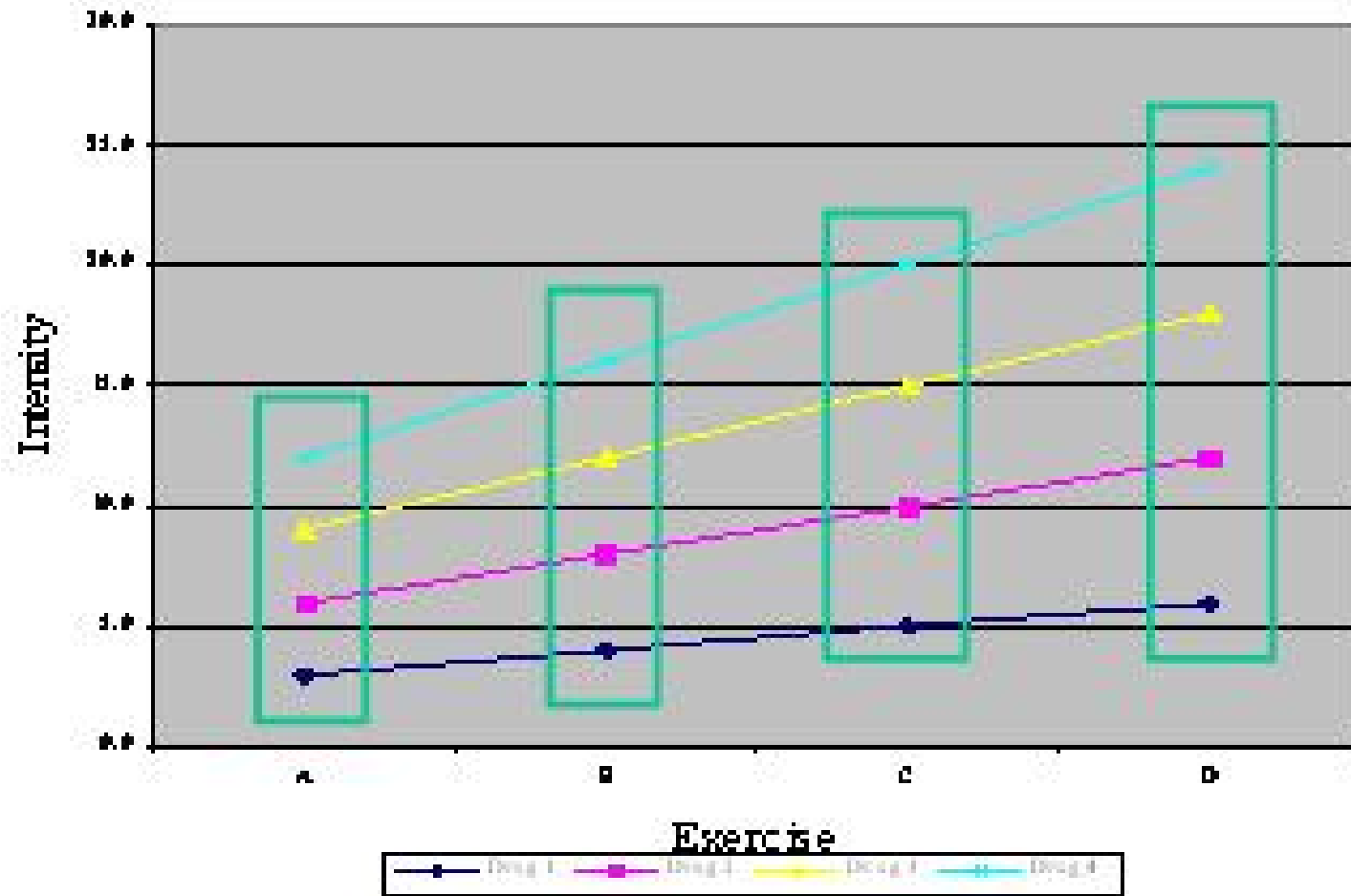
Interaction is present when the differences between the levels for one factor are different for the different levels of the other factor. *That is, when differences are different.*

	Treatment Combination Means				
	Exercise				Drug Means
Drug	A	B	C	D	
1	4.67	1.67	6.33	3.33	4.00
2	4.00	5.00	3.33	5.33	4.42
3	4.67	5.67	5.33	6.67	5.58
4	2.33	6.00	5.00	7.33	5.17
Exercise Means	3.92	4.58	5.00	5.67	4.79

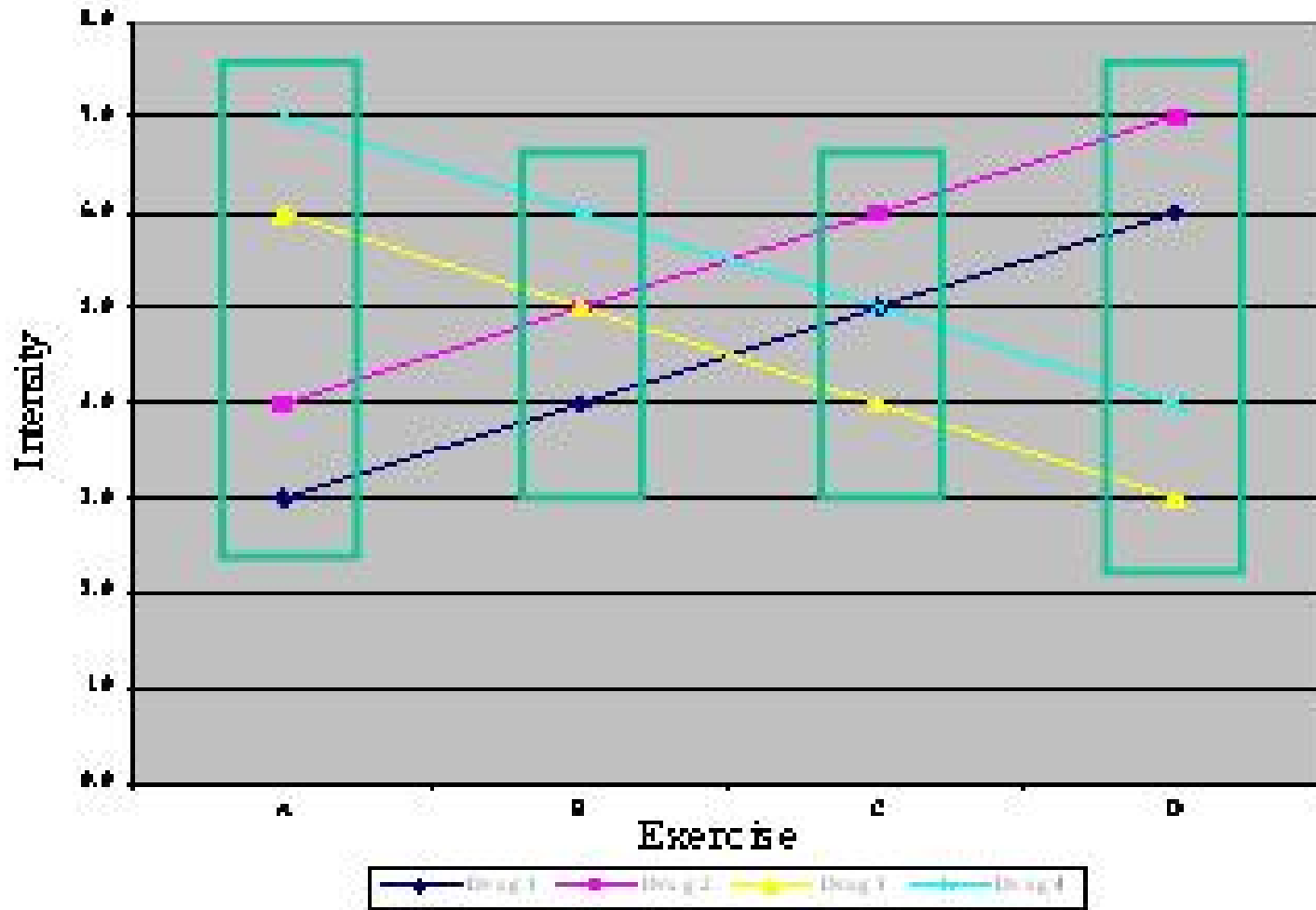
No Interaction Between Exercise and Drug



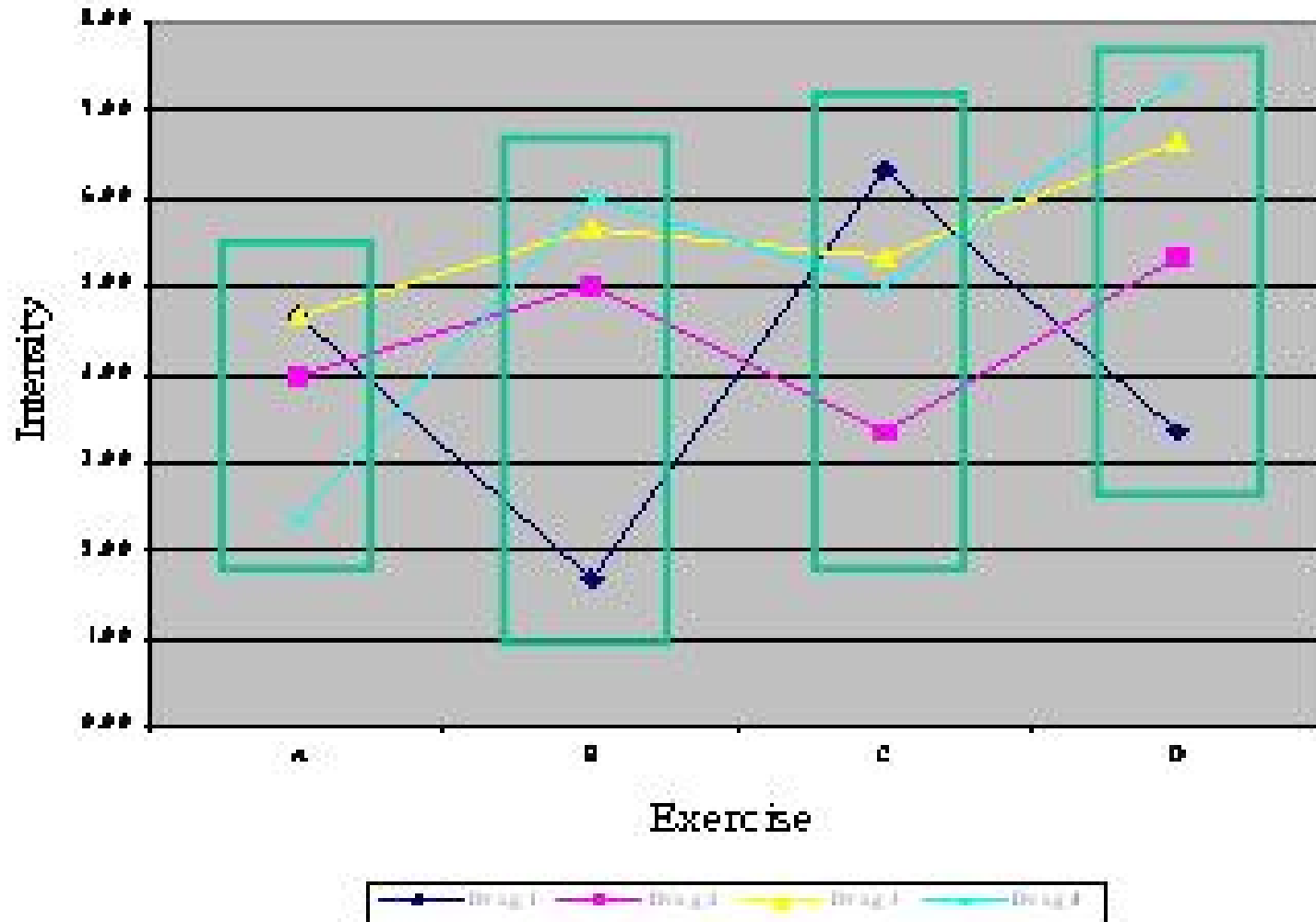
Subtle Interaction



Dramatic Interaction



Interaction from Example



Hypotheses to Test

First, test:

H_0 : No Interaction

If this hypothesis is not rejected, then test:

H_0 : No Exercise effect, and

H_0 : No Drug effect

When There is Interaction

If the null hypothesis of no interaction is rejected, then a test for overall drug effect or overall exercise effect has no meaning. Under these circumstances, these tests should not be done; however, it is likely to be insightful to examine the patterns of interaction between the two factors.

Hypothesis Testing

1. The hypotheses:

First test:

H_0 : No Interaction

Then, if there is no interaction, test:

H_0 : No Exercise effect, and

H_0 : No Drug effect

2. The Assumptions:

Independent random samples, normal distributions, equal variances

3. The α level:

$$\alpha = 0.05$$

4. and 5. Test Statistic and Rejection Region

See ANOVA table

6. The Test Result:

ANOVA as shown in the following slides

Activity response score

Drug	Type of Exercise				Total
	A	B	C	D	
1	4	2	9	3	
	3	2	7	4	
	7	1	3	3	
Total	14	5	19	10	48
2	2	5	5	5	
	4	3	2	4	
	6	7	3	7	
Total	12	15	10	16	53
3	3	4	7	9	
	6	8	5	5	
	5	5	4	6	
Total	14	17	16	20	67
4	2	9	4	9	
	2	4	6	5	
	3	5	5	8	
Total	7	18	15	22	62
Overall	47	55	60	68	230

Note: $230^2/48 = 1,102.08$

SS(Total)

Drug	Individual values squared				Total
	A	B	C	D	
1	16	4	81	9	110
	9	4	49	16	78
	49	1	9	9	68
2	4	25	25	25	79
	16	9	4	16	45
	36	49	9	49	143
3	9	16	49	81	155
	36	64	25	25	150
	25	25	16	36	102
4	4	81	16	81	182
	4	16	36	25	81
	9	25	25	64	123
Table Total					1,316

$$SS(\text{Total}) = 1,316 - 1,102.08 = 213.92$$

$$\text{Note: } 230^2/48 = 1,102.08$$

SS(Treatment Combinations)

The next step is to prepare the following table showing the sums for each of the 16 treatment combinations. Also shown are the factor level means.

	Cell Sums					
	A	B	C	D	Total	Mean
1	14	5	19	10	48	4.0
2	12	15	10	16	53	4.4
3	14	17	16	20	67	5.6
4	7	18	15	22	62	5.2
Total	47	55	60	68	230	
Mean	3.9	4.6	5.0	5.7		4.8

More for SS(Treatment Combinations)

	$(\text{Cell Sum})^2/3$				
	A	B	C	D	Total
1	65.33	8.33	120.33	33.33	227.33
2	48.00	75.00	33.33	85.33	241.67
3	65.33	96.33	85.33	133.33	380.33
4	16.33	108.00	75.00	161.33	360.67
Total	195.00	287.67	314.00	413.33	1,210.00

$$\begin{aligned} \text{SS(Treatment Combinations)} &= 1,210.00 - 1,102.08 \\ &= 107.92 \end{aligned}$$

Note: $230^2/48 = 1,102.08$

We will call this SS(Treatments) afterward to save space.

SS(Within Cells)

$$\begin{aligned}SS(\text{Within Cells}) &= SS(\text{Total}) - SS(\text{Treatments}) \\ &= 213.92 - 107.92 \\ &= 106.00\end{aligned}$$

SS(Exercise)

	Cell Sums						
	Exercise						
Drug	A	B	C	D	Total	Mean	Total ² /12
1	14	5	19	10	48	4.0	192.0
2	12	15	10	16	53	4.4	234.1
3	14	17	16	20	67	5.6	374.1
4	7	18	15	22	62	5.2	320.3
Total	47	55	60	68	230		1,120.50
Mean	3.9	4.6	5.0	5.7		4.8	
Total ² /12	184.1	252.1	300.0	385.3	1,121.50		

$$SS(\text{Exercise}) = 1,121.50 - 1,102.08 = 19.42$$

$$\text{Note: } 230^2/48 = 1,102.08$$

SS(Drugs)

Drug	Cell Sums				Total	Mean	Total ² /12
	Exercise						
	A	B	C	D			
1	14	5	19	10	48	4.0	192.0
2	12	15	10	16	53	4.4	234.1
3	14	17	16	20	67	5.6	374.1
4	7	18	15	22	62	5.2	320.3
Total	47	55	60	68	230		1,120.50
Mean	3.9	4.6	5.0	5.7		4.8	
Total ² /12	184.1	252.1	300.0	385.3	1,121.50		

$$SS(\text{Drugs}) = 1,120.50 - 1,102.08 = 18.42$$

$$\text{Note: } 230^2/48 = 1,102.08$$

SS(Drug x Exercise Interaction)

$$\begin{aligned}SS(\text{Interaction}) &= SS(\text{Treatments}) - SS(\text{Drugs}) \\ &\quad - SS(\text{Exercise}) \\ &= 107.92 - 18.42 - 19.42 \\ &= 70.08\end{aligned}$$

ANOVA					
Source		df	SS	MS	F
Exercise		3	19.42	6.47	1.95
Drug		3	18.42	6.14	1.85
Interaction		9	70.08	7.79	2.35
Treatments		15	107.92	7.19	
Within		32	106.00	3.31	
Total		47	213.92		

$$F(\text{Interaction}) = 7.79/3.31 = 2.35 \quad F_{0.95}(9, 32) = 2.19$$

$$F(\text{Exercise}) = 6.47/3.31 = 1.95 \quad F_{0.95}(3, 32) = 2.90$$

$$F(\text{Drugs}) = 6.14/3.31 = 1.85 \quad F_{0.95}(9, 32) = 2.90$$

7. The Conclusion:

Reject H_0 : No Interaction since the F statistic calculated from the ANOVA table, $F = 2.35$ exceeds $F_{0.95}(9, 32) = 2.18$

Do not test the other two hypotheses.

Variations and Review

The following slides present three different designs, but use the same data. This is done for the dual purpose of reviewing the different types of ANOVA we have discussed, while also showing their relationship to each other.

The data are an activity intensity score.

Variation 1: One-Way ANOVA

	Drugs			
	A	B	C	Total
	4	7	9	
	3	9	7	
	5	3	6	
	2	5	5	
	4	3	8	
	3	7	7	
Sum	21	34	42	97
Mean	3.5	5.7	7.0	5.4

$$SS(\text{Drugs}) = 21^2/6 + 34^2/6 + 42^2/6 - 97^2/18$$

ANOVA				
Source	df	SS	MS	F
Drugs	2	37.44	18.72	6.26
Within	15	44.83	2.99	
Total	17	82.28		

$$F(\text{Drugs}) = 6.26 \quad F_{0.95}(2, 15) = 3.68$$

Variation 2: Randomized Blocks, Simple Repeated Measures

Person	A	B	C	Total	Mean
1	4	7	9	20	6.7
2	3	9	7	19	6.3
3	5	3	6	14	4.7
4	2	5	5	12	4.0
5	4	3	8	15	5.0
6	3	7	7	17	5.7
Sum	21	34	42	97	
Mean	3.5	5.7	7.0		5.4

$$SS(\text{Drugs}) = 21^2/6 + 34^2/6 + 42^2/6 - 97^2/18$$

$$SS(\text{Persons}) = 20^2/3 + 19^2/3 + \textcircled{1} + 17^2/3 - 97^2/18$$

ANOVA				
Source	df	SS	MS	F
Drugs	2	37.44	18.72	6.41
Person	5	15.61	3.12	1.07
Residual	10	29.22	2.92	
Total	17	82.28		

$$F(\text{Drugs}) = 6.41 \quad F_{0.95}(2, 10) = 4.10$$

$$F(\text{Persons}) = 1.07 \quad F_{0.95}(5, 10) = 3.33$$

Variation 3: Factorial Design

Exercise	A	B	C	Total
1	4	7	9	
	3	9	7	
Total	7	16	16	39
2	5	3	6	
	2	5	5	
Total	7	8	11	26
3	4	3	8	
	3	7	7	
Total	7	10	15	32
Sum	21	34	42	97
Mean	3.5	5.7	7.0	5.4