Is it possible that this sample with $\bar{x} = 12.6$ *came from a population with* $\mu = 13.0$?

Is it likely that this sample with $\bar{x} = 12.6$ *came from a population with* $\mu = 13.0$?

For $\alpha = 0.01$ for the first of the random samples of size $n = 5$ from the population of body weights, we had

$n = 5$
$\bar{x} = 153.0$
$\sigma = 10$

$$C\left[\bar{x} - 2.576 \; \frac{10}{\sqrt{5}} \le \mu \le \bar{x} + 2.576 \; \frac{10}{\sqrt{5}}\right] = 0.99$$

becomes

$$C\left[153.0 \;\; - 2.576 \; \frac{10}{2.23} \le \mu \le 153.0 + 2.576 \; \frac{10}{2.23}\right] = 0.99$$

or

$$C\left[153.0 - 2.576\left(4.47\right) \le \mu \le 153.0 + 2.576\left(4.47\right)\right] = 0.99$$

or

$$C\left[153.0 - 11.51 \le \mu \le 153.0 + 11.51\right] = 0.99$$

or

$$C\left[141.5 \le \mu \le 163.1\right] = 0.99$$

For $\alpha = 0.01$ for the first of the random samples of size n = 20 from the population of body weights, we had

$n = 20$
$\bar{x} = 151.6$
$\sigma = 10$

$$C\left[\bar{x} - 2.576\ \frac{10}{\sqrt{20}} \leq \mu \leq \bar{x} + 2.576\ \frac{10}{\sqrt{20}}\right] = 0.99$$

becomes

$$C\left[151.6 - 2.576\ \frac{10}{4.47} \leq \mu \leq 151.6 + 2.576\ \frac{10}{4.47}\right] = 0.99$$

or

$$C\left[151.6 - 2.576(2.23) \le \mu \le 151.6 + 2.576(2.23)\right] = 0.99$$

or

$$C\left[151.6 - 5.74 \le \mu \le 151.6 + 5.74\right] = 0.99$$

or

$$C\left[145.9 \le \mu \le 157.3\right] = 0.99$$

Confidence Intervals: Confidence intervals for the population mean μ, are an estimation procedure with reasonable bounds about the sample mean $\bar{x}$,

In general, the closer the bounds are to the point estimate, $\bar{x}$, the better the point estimate.

The bounds are constructed in a manner that takes into account the variability of the point estimate.

The bounds are also based on an appropriate probability distribution so that some reasonable probability statements can be made.

Our confidence is in the process we used to generate a specific confidence interval and not in the specific interval itself.

In general, we construct such intervals so that, should we repeat the process a large number of times, then 95%, for a 95% confidence interval, of such intervals should contain the population parameter being estimated by the point estimate and the confidence interval.

The specific interval we compute in any given situation may or may not contain the population parameter.

The only way for us to be sure that the population parameter is within the bounds of the confidence interval is to know the true value for this parameter.

Obviously, if we knew the true value, we would not bother to go through the process of guessing at the truth with estimates.

AJPH Example--Mean, SD, Variance, CI

In the following example (Table 1, Elola et al, AJPH, 1995, 85: p1398), the mean and SD for health expenditures per capita (US$) for the $n = 7$ countries with Social Security Systems are given.

What is your best guess as to the population mean for the population from which this sample was selected?

What is the sample variance for the individual measures of health expenditures in this sample of size n = 7?

Construct a 95% confidence interval for this population mean. For this calculation, assume that the population variance is \$22,500 (dollars^2) and that the population standard deviation is \$150.

What is the variance and standard deviation for the population of means of all possible samples of size n = 7 for this situation?

TABLE 1—Mean Values for Health Indicators and Socioeconomic Variables for Western European Countries with Social Security Systems and National Health Service Systems

	Social Security Systems (n = 7)		National Health Services (n = 10)		
	Mean	SD	Mean	SD	P
Gross domestic product per capita (US$)	17 799.9	1 866.2	14 281.3	3 184.3	< .01
Health care expenditures per capita (US$)	$\boxed{1\ 427.6}$	142.5	1 057.5	334.1	< .01
Population covered by the system, %	93.9	11.3	99.9	0.3	NS
Public health expenditures as % of total health expenditures	76.1	9.9	81.9	5.8	NS
Gini coefficient	0.3	0.04	0.3	0.04	NS
Infant mortality rate (deaths/1000 live births)	7.3	0.4	7.4	1.3	NS
PYLL, females	2 812.6	296.6	2 833.7	285.1	NS
PYLL, males	5 312.7	630.9	4 928.0	614.3	NS
Life expectancy, males, y	72.6	1.1	73.2	1.5	NS
Life expectancy, females, y	79.5	0.9	79.1	1.2	NS

Note. NS = not significant; PYLL = potential years of life lost.

A Confidence Interval and a Question

Suppose we have a random sample of n = 25 measurements of chest circumference from a population of newborns with $\sigma = 0.7$ in. The sample mean is $\bar{x} = 12.6$ in.

A 95% confidence interval for μ is:

$$C[\bar{x} - 1.96\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.96\sigma/\sqrt{n}] = 0.95$$

$$C[12.6 - 1.96(0.7/5) \le \mu \le 12.6 + 1.96(0.7/5)] = 0.95$$

$$C[12.6 - 0.27 \le \mu \le 12.6 + 0.27] = 0.95$$

$$C[12.33 \le \mu \le 12.87] = 0.95$$