

Module 24: One-sided, One-tailed Tests

This module discusses the difference between two-sided and one-sided hypothesis tests.

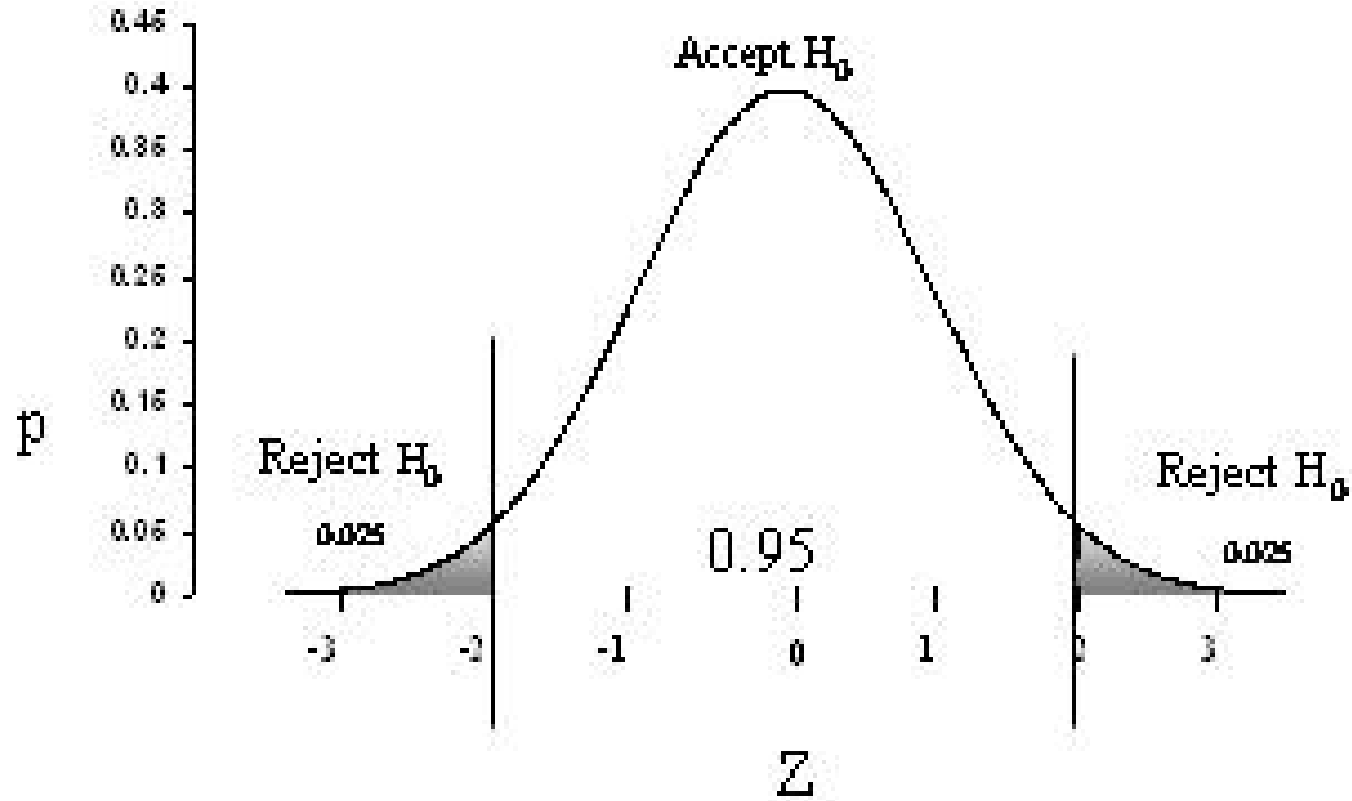
One-Sided or One-Tailed Hypothesis Tests

In most applications, a two-sided or two-tailed hypothesis test is the most appropriate approach. This approach is based on the expression of the null and alternative hypotheses as follows:

$$H_0: \mu = 170 \text{ vs } H_1: \mu \neq 170$$

To test the above hypothesis, we set up the rejection and acceptance regions as next slide, where we are using $\alpha = 0.05$:

Normal Distribution



In this example, the rejection region probabilities are equally split between the two tails, thus the reason for the label as a two-tailed test.

This procedure allows the possibility of rejecting the null hypothesis, but does not specifically address, in the sense of statistical significance, the direction of the difference detected.

There may be situations when it would be appropriate to consider an alternative hypothesis where the directionality is specifically addressed. That is we may want to be able to select between a null hypothesis and one that explicitly goes in one direction. Such a hypothesis test can best be expressed as:

$$H_0: \mu = 170 \text{ vs } H_1: \mu > 170$$

The expression is sometimes given as:

$$H_0: \mu \leq 170 \text{ vs } H_1: \mu > 170$$

The difference between the two has to do with how the null hypothesis is expressed and the implication of this expression.

The first expression above is the more theoretically correct one and carries with it the clear connotation that an outcome in the opposite direction of the alternative hypothesis is not considered possible.

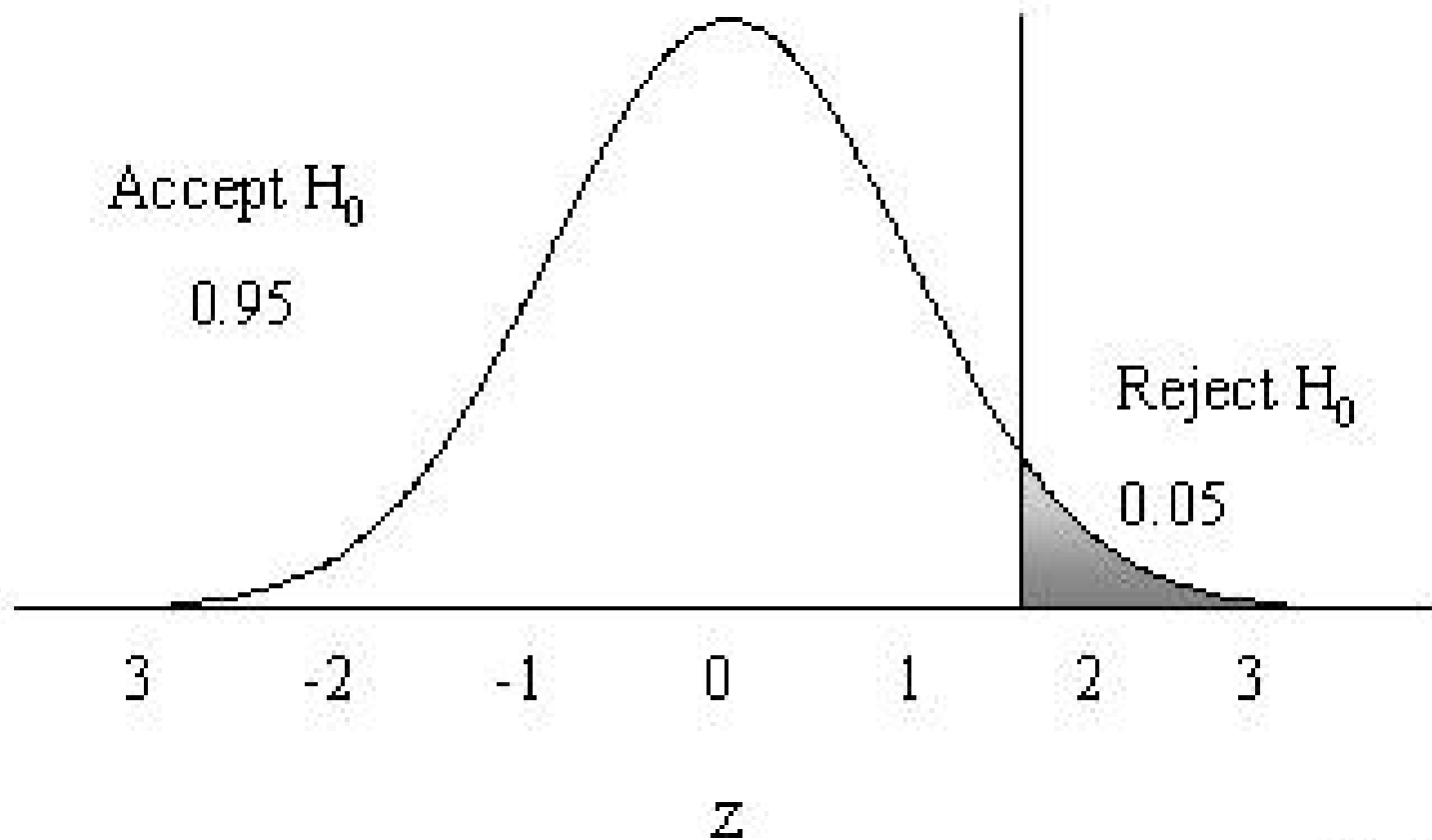
This is, in fact, the way the test is actually done.

The process of testing the above hypothesis is identical to that for the two-tailed test except that all the rejection region probabilities are in one tail.

For a test, with $\alpha = 0.05$, the acceptance region would be, for example, the area from the extreme left up to the point below which lies 95% of the area.

The rejection region would be the 5% area in the upper tail.

Normal Distribution



It should be noted that a smaller value of the test statistic will lead to the rejection of the null hypothesis for a one-sided test than is the case for a two-sided test.

This circumstance sometimes leads to abuse of the use of the one-sided test because it is easier to obtain statistical significance with it.

For this reason and because there are few circumstances in health and medical research where the possibility of an outcome in the undesired direction can be ruled out completely, the one-sided test should be used with caution.

For those circumstances for which it is appropriate, it can be a useful test.